The Jewish Calendar's Molad System

The Jewish calendar is luni-solar; its months are lunar, but its years are solar. By this, we mean that the calendar's mean year-length is fairly close to the length of a solar year, so its years stay (approximately) in step with the seasons. A year begins, on average, around the time of the September (southward) equinox, and Passover, which occurs just after mid-year, occurs shortly after the March (northward) equinox. The calendar's months, on the other hand, are synchronised with the phases of the Moon. Each month begins fairly close to an astronomical New Moon (dark moon) and Full Moon coincides approximately with the middle of the month. This is because the calendar's mean month length is very close to the mean length of what astronomers call a **synodic lunation** (a full cycle of lunar phases).

THE SYNODIC LUNATION

The illumination cast upon Earth by the Moon is not the Moon's own light; moonshine is sunlight reflected off the Moon's surface toward the Earth. At all times exactly half of the Moon (one hemisphere) is illuminated by the Sun, but, depending on the positions of the Sun and the Moon as seen from Earth, which are constantly changing as the Moon orbits the Earth, only some, none or all of its sunlit side is facing Earth.

As the amount of the Moon's illuminated side that is visible to us changes, the Moon appears to change shape, growing and then shrinking in succession. This is what we mean by a cycle of lunar phases. The word **synodic** comes from the Greek word for meeting. It is used to describe this cycle, also called the Moon's synodic cycle, because it notionally begins at **lunar conjunction**, when the Sun and Moon appear from Earth to meet at the same celestial longitude.

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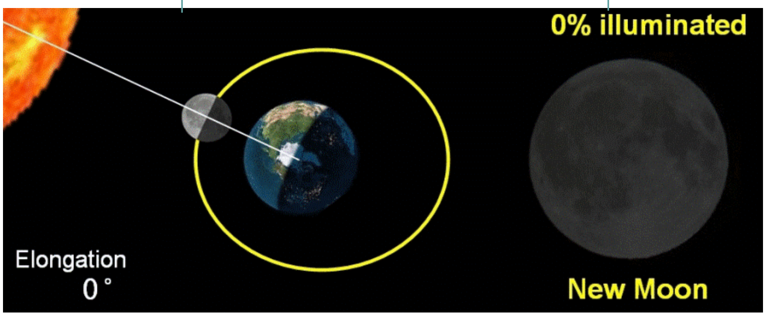

This beginning point is when the Sun, Moon and Earth are in line with one another in that order so that the Moon is between us and the Sun. At that point in its orbit, the Moon's sunlit side is its far side (the side facing away from us). All of its near side (the side facing us) is dark at that time so we cannot see the Moon at all. That is the astronomical **New Moon** phase.

Over the next two weeks the Moon goes through the **waxing** (growing) half of its cycle, first appearing, one or two evenings after New Moon, as a thin crescent (, then as a half-moon shape ◖ , then as a gibbous shape ●(concave on both sides but not fully circular), until, halfway through the cycle, it appears as a Full Moon ●. Then for another two weeks it goes through the **waning** (shrinking) half of its cycle, until, about a day before the next New Moon, it disappears from view again. For about a day either side of New Moon, as the Moon approaches and then departs conjunction, it is too close in our sky to where the Sun is to be visible to us. Astronomers divide this cycle into eight phases:
New Moon ◯, Waxing Crescent (, First Quarter ◖ , Waxing Gibbous ● ,
Full Moon ●, Waning Gibbous ●, Last Quarter ◗, Waning Crescent). { (Depictions shown for Southern Hemisphere.)
These are global, not local, phenomena; they occur at the same moment for everyone on Earth.

Because the Moon's orbit is elliptical, not circular, it does not orbit the Earth at a uniform speed. Its orbital speed increases the closer it comes to Earth and decreases as it recedes from the Earth. Therefore real lunations vary in length, but the *mean* length of a synodic lunation is about 29.53 days.

THE MOLAD SYSTEM AND THE METONIC CYCLE

The Jewish calendar of today is a rule based, fixed-arithmetic calendar. It evolved from an earlier model (also luni-solar) that was regulated by observation combined, increasingly, with astronomical modelling.

Its months are kept in step with the lunar phases by structuring the calendar around moladot, which are *mean* lunar conjunctions, i.e. calendric approximations of New Moons that occur at fixed intervals. That molad interval is a *calendric* lunation and its length is constant. So the calendric moladot and lunations model the real astronomical ones, but they correspond only approximately to the real ones, because real lunations vary in length for the reason given above. (Molad is short for מולד הלבנה, lunar conjunction.)

The calendar's lunations also correspond to its months. This correspondence too is approximate because a calendric lunation is about 29.53 days, whereas a calendar month must have a whole number of days.

To create this correspondence the calendar alternates long months of 30 days with short months of 29 days. Some variation is allowed to this alternation of long and short months, with the result that long months outnumber the short ones in a ratio of about 53% to 47%. This makes the calendar's mean month-length about 29.53 days. A month always begins on the day of a calendric molad or within the next n days, where n=2 for month 1, Tishrei, and 3 for all other months. The **molad of a year** means the molad near the beginning of the year, i.e. near the beginning of Tishrei.

Twelve such months equals about 354 days, which is nearly 11 days shorter than a solar year, so every so often an extra month is intercalated (inserted into the calendar), creating a leap year of 13 months. This increases the calendar's mean year-length to approximately that of a solar year, thereby keeping its lunar months roughly in step with the seasons.

Nineteen solar years is almost equal in length to 235 synodic lunations. (It falls short of the latter by only two hours.) This correlation was first used (at least in the western world) by the Athenian astronomer Meton in 432 BCE in his reformation of the Greek luni-solar calendar. He put in place a scheme for regulating its leap years using a cycle of 19 years, of which seven were leap years of 13 months and the other twelve were common years of 12 months. Thus, each Metonic cycle (the name by which it has been known ever since) has 235 months (19×12 +7).

More than seven centuries later, Meton's leap year scheme was adopted by the Jewish calendar when it too was undergoing a transformation in the mid fourth century CE. By that time however, a better approximation than Meton's for the mean length of a synodic lunation (**L**) had been found by the Greek astronomer Hipparchus in 146 BCE. The Jewish calendar adopted Hipparchus's value of 29.5 days, 44 minutes and 3 and 1/3 seconds for L and 235L/19 for its mean year-length. This value for L is the exact value of the calendar's fixed molad-interval (the length of a *calendric* lunation) and hence the calendar's mean month-length. This is the value stated above approximately as 29.53 days.

NYCTHEMERON, NOCTDIEM AND THE JEWISH DAY (יממה)

The Hebrew word y^{e}mama (יממה) means a twenty-four-hour day. The equivalent English term is **nycthemeron**. In the civil calendar, a calendar day is a nycthemeron commencing at midnight. Here, I will be using the term **noctdiem** for a particular type of nycthemeron, one consisting of an entire night plus the whole of the adjacent daylight period – either the one preceding the night or the one following the night. The latter type of noctdiem is a calendar day of the Jewish calendar and it commences at mean sunset.

This property of the Jewish calendar is shared by nearly all lunar and luni-solar calendars. This is no coincidence. In nearly every calendar whose months are lunar a new month begins (or, even if the rules have changed, it once began) at the first appearance, after lunar conjunction, of the waxing lunar crescent. The Moon orbits the Earth from west to east, and at that time in the Moon's synodic cycle it is only about 10° east of the Sun, so this phenomenon is always seen low in the western sky *just after sunset*. This is why, in such calendars, the calendar day begins at sunset or at mean sunset.

A notable exception to this (perhaps the only one) is the ancient Egyptian lunar calendar. (This is the one that was used for religious purposes; it operated concurrently with a solar calendar used for civil purposes.) In that lunar calendar, the calendar day was the other type of noctdiem, an entire daytime period plus the whole of the following night. This is because a month in that calendar began with the *last* appearance of the *waning* lunar crescent (the 'old' moon). At that time in the Moon's synodic cycle it is about 10° *west* of the Sun, so this phenomenon is always seen low in the *eastern* sky *just before sunrise*. So in that calendar, the calendar day began at sunrise.

JEWISH TIME

In Jewish sources, all time values specified in calendric rules and used in calendric calculations are expressed in **Jewish Mean Time** (explained below), using 24-hour clock notation. Thus, times are expressed as hh:pppp, where hh = 0 to 23 hours and pppp = 0 to 1079 parts of an hour. (Parts are the Jewish divisions of an hour, where 1 hour = 1080 parts, 1 minute = 18 parts, and 1 part = 3 and 1/3 seconds.) Zero hours is at mean sunset (18:00 hours, civil time).

All moladot are expressed as a time of week, i.e. weekday W at time T, in Jewish Mean Time, and the time component is expressed in the format specified just above.

JEWISH MEAN TIME (JMT) is the time of day where the hours of the day are numbered (from 0 to 23) commencing from when a new Jewish day begins for calendric purposes. A Jewish calendar-day consists of an entire night plus the whole of the following daylight period. For calendric purposes, a new Jewish day begins at mean sunset, exactly six hours before the corresponding civil day begins at midnight. So JMT (everywhere) = (your local) civil time (CT) plus six hours, e.g. Friday, 18:00, CT = Saturday, zero hours, JMT. JMT is used for all calendric purposes and all associated time reckoning.

The last 18 hours of the Jewish day coincide with the first 18 hours of the corresponding civil day.

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The "mean" in JMT signifies that all hours are 'standard hours,' which are exactly 60 minutes long. (Sixty minutes is one 24th of the mean interval between two (real) noons.)

This is in contrast to **JST**, which is a different Jewish system for specifying times. It is similar to local solar time (AKA sundial time). In this system a new day begins at real (not mean) sunset and times of the day and night are relative to local sunrise and sunset. It uses 'seasonal hours,' which are of variable length. A seasonal hour of the day is one twelfth of the daytime, so it is proportional to the length of daytime, and a seasonal hour of the night is one twelfth of the night-time, so it is proportional to the length of the night. So a daytime hour usually differs in length from an hour of the night and both vary in length depending on the season and your latitude. Only around the time of the two equinoxes, or all year round at the equator, are these hours exactly 60 minutes.

Seasonal hours are used for specifying the times of religious observances that apply to specific days or to daytime only or night-time only or to particular portions of the day or night. Examples would be Shabbat observance and prayer times.

THE EPOCH OF THE JEWISH CALENDAR, MOLAD TOHU

It took many centuries for the Jewish calendar to evolve into its present day form. It did not adopt all the rules in use today until around the tenth century CE. However it nowadays numbers its years according to a chronology, which, based on a literal reading of the Bible, regards year 1 of the calendar to be the year of creation. That year corresponds to BCE year 3760 (Julian proleptic and Gregorian proleptic).

Dates in a calendar that precede the introduction of that calendar are said to be **proleptic** (meaning anticipatory). They are arrived at by projecting the calendar backwards from the time of its introduction to the subject date, applying that calendar's rules retrospectively to the whole of the intervening period. Proleptic dates are therefore theoretical rather than historical.

The epoch of the present-day Jewish calendar is Monday, 1 Tishrei, of Jewish year 1. (BCE 3761, 7 Oct, Julian proleptic, 7 Sep, Gregorian proleptic.) Epoch in this context means the starting point of a calendar or a calendar era. This epoch is proleptic because it not only predates the current Jewish calendar, it also predates the Jewish people. Also, the system of year numbering now in use was not adopted until well after the Jewish calendar attained its present form. Prior to the adoption of the year-numbering in use today, various chronological eras were used by Jews at different times and places.

Notwithstanding what was mentioned above about what we nowadays count as Jewish year 1 being regarded as the year of creation, the above epoch is not when the world was held, by that belief, to have been created, although the contrary is often erroneously stated. According to that chronology, the creation of the world took place in the *last week* of year 1. The Friday of that week is held to have been the sixth day of creation, the day that Adam was made in the biblical creation story. That day coincides with a proleptic molad Tishrei of the present-day calendar.

Therefore that chronology counts the first day of Adam's life as the beginning of year 2 and it counts the previous days of creation (which must belong to some year) as belonging to the last week of year 1. According to that belief, since most of year 1 (all but its last few days) predates creation, that year is mostly theoretical (just a mathematical construct). It was therefore named in rabbinic literature "the year **Tohu**," from the word in Genesis 1:2 describing the amorphous and vacuous state of the world at the beginning of its creation. Nevertheless, it is the *beginning* of year 1 (not its last week) that is used as the epoch of the present-day calendar. More precisely, the epoch is the (theoretical) molad Tishrei of that year. That molad is called **Molad Tohu**, after the name given to the year. (That Tishrei and Tishrei of year 2 are called תשרי של תוהו and תשרי של יישוב in rabbinic literature.)

Molad Tohu was originally determined (by backward reckoning) to have occurred on Monday (1st Tishrei), at 05:0204 (hh:pppp), Jewish Mean Time. In the civil calendar that is 23:11:06 (hh:mm:pp) on Sunday, 6 October, Julian proleptic (6 Sep, Gregorian proleptic), 3761 BCE. The Julian Day Number at noon of that Sunday = 347,997. That molad is the calendar's (proleptic) epoch for all molad calculations. It is also known as molad BHRD (ב, ה, ר"ד). That name is a Hebrew mnemonic for the time of its occurrence, formed from the Hebrew numerals for 2 (Monday), 5, 204. The mnemonic is used because it is a fundamental quantity of the present-day calendar – the base molad from which all subsequent moladot of the calendar can be computed by adding to it the appropriate number of molad intervals as explained below.

CALCULATION OF MOLADOT AND THE CALENDAR OF A YEAR

We mentioned above that all moladot are expressed as a time of week, i.e. weekday W at time T, in Jewish Mean Time. Weekday W is never ambiguous because the molad of a month always occurs on the first of the month or within the previous three days.

The calculation of moladot is essential to determining the **year type** of a year, Y, which dictates which of 14 possible year calendars applies to year Y. A year type may be fully specified by two properties of the year: the weekday on which it commences and its length in days. The allowable values for these two properties are shown in Tables 1 and 2 below. As will be seen from Table 1, the middle digit (5 or 8) of the year length indicates whether Y is a common year (שנה פשוטה) or a leap year (שנה מעוברת) and the last digit (3, 4 or 5) indicates the year's form, i.e. whether it is short, regular or long (חסרה \ כסדרה \ שלמה).

Table 1: The six permitted year lengths and how they affect the lengths of Heshvan & Kislev

	Year form		
	Short	Regular	Long
Days in Heshvan	29	29	30
Days in Kislev	29	30	30
Common Year Length	353	354	355
Leap Year Length	383	384	385

Table 2: The 14 permitted year types (Keviot)

Commencement Day	Year Length (in days)					
	Common			Leap		
	353	354	355	383	384	385
Monday	We	~~Th~~	Fr	Fr	~~Sa~~	Su
Tuesday	~~Th~~	Fr	~~Sa~~	~~Sa~~	Su	~~Mo~~
Thursday	~~Sa~~	Su	Mo	Mo	~~Tu~~	We
Saturday	Mo	~~Tu~~	We	We	~~Th~~	Fr

In Table 2, forbidden year types (explained below) are those whose cells are shaded and have a diagonal strike across them. The other 14 are the permitted year types. The two-letter, abbreviated weekday names in those cells is the weekday on which a year of that type would end (and also the weekday of day 2 of Passover).

The year type of Y is determined from the computed values of the two moladot at the beginning and end of Y and from the calendar rules known as the four **postponements** (דחיות) that govern whether a year will commence on the day of its molad or whether its commencement will be postponed by one or two days. This must be determined for both year Y and Y+1 to establish the weekdays on which both will commence. Once that is known, the following arithmetic procedure will give you the year length of Y.

Assign the values 1 to 7 to the weekdays Sunday to Saturday. Using those values, let W_1 and W_2 be, respectively, the weekday on which Y and Y+1 commences. Let $D = W_2 - W_1$. If $D < 1$, add 7 to it. D is now the number of days by which the year length exceeds n whole weeks (where n=50 for a common year and 54 for a leap year). The year length of Y can now be determined from Table 3.

Table 3: D gives year length

D =	Common Year	D =	Leap Year
3	353	5	383
4	354	6	384
5	355	7	385

It is also necessary to know, for each of the years Y−1, Y and Y+1, whether the year is a common year (שנה פשוטה) or a leap year (שנה מעוברת). This can be determined from the remainder (R) of Y/19. If R = 0, 3, 6, 8, 11, 14 or 17, Y is a leap year. From Table 3 it is obvious why this is relevant to year Y. It is also relevant to all three years for the additional reason that it is one of the criteria for determining whether postponement rules 3 and 4 will affect the commencement of year Y or Y+1, as will be seen below.

If the commencement of a year, Y, is postponed, it has the effect of lengthening year Y−1 and/or shortening year Y. This is the reason for the year-form variations shown in Table 1.

A METHOD FOR CALCULATING A MOLAD

This is based on a very simple principle: Since the molad interval (**L**) is constant, if the time-of-week (**T**) of any molad (M_1) is known, the time of any subsequent molad (M_2), occurring **n** months later, is T + nL.

The convention of specifying a molad only as a time of week makes the calculation easier by allowing whole weeks to be removed from all the terms used and from all the intermediate results and the final result. This removal does not affect the calculation because a time of week plus or minus x whole weeks is the same time of week.

For this purpose the usual practice is to use Molad Tohu for M_1. This is the first molad of the calendar, molad Tishrei of year 1. This has two advantages:

(a) the value of Molad Tohu is well known to Jewish calendrists, who remember its value by the traditional Hebrew mnemonic, BHRD (ב, ה, ר"ד), attached to it. The mnemonic is formed from the Hebrew numerals used in the notation of its value, Monday (day 2), 05:0204.

(b) if the desired molad (M_2) is the first molad of a year (the molad of Tishrei), the value of **n** is easily determined from the year number, Y, by the following formula, which gives the number (n) of whole months from the beginning of year 1 to the beginning of Y.

n = (235Y – 234) \ 19 (This is the sum of: [12(Y−1)] + [(7(Y−1)+1)\19]. See explanation below.)

where \ denotes an integer division, meaning the remainder is discarded and only the integer quotient is retained.

(If you want the number of months from the beginning of year 1 up to the beginning of month M of year Y, where M > 1, just add M−1 to the result obtained above for n.)

Using the method described above, the formula for the molad of month M of year Y is:

{T + {n × {L}}}, where:

T = the time-of-week of Molad Tohu = Monday, 05:0204.

n is the number of months from the beginning of year 1 to the beginning of month M of year Y, obtained by the first formula given above.

L = the molad interval = 4 weeks, 1 day, 12 hours, 793 parts.

{X} = X modulo 7 days, i.e. the time quantity X after eliminating all whole weeks contained in X.

The three sets of these braces in the formula signify that:

(a) the value L is reduced by 4 weeks, then
(b) the product nL is reduced by as many whole weeks as are contained in that product, and
(c) if the sum of that reduced product plus T exceeds one whole week, it is reduced by one week.

THE FOUR POSTPONEMENT RULES

These rules are known by mnemonic names derived from the Hebrew numerals used in their specification.

(1) ADU (אד"ו). This prevents a year from beginning on a Sunday, Wednesday or Friday, JMT, which are weekdays 1, 4 and 6.

(2) YaCH (י"ח). This dictates that if the molad at the beginning of a year occurs at or after mean noon, which, in JMT, is 18:0000 hours (hh:pppp), it is called an aged molad (מולד זקן) and the commencement of the year is postponed to the next permitted day, which may require a postponement of two days. This is required if the day of the aged molad immediately precedes a day forbidden by ADU. The two day postponement in such cases is called **YaCH-ADU**.

Postponement rules 3 and 4 are just mathematical consequences of rules 1 and 2 plus the restriction that the permitted year lengths are only those six listed in Table 1. This will be explained in more detail below. Cases of postponements 3 and 4 seldom occur. They account for only 5.43% and 0.88%, respectively, of all cases of postponement. Those rules are:

(3) GaTRaD (ג, ט, ר"ד). If the molad at the beginning of a common year occurs on Tuesday (יום ג), at or after 09:0204 (hh:pppp), JMT, the year commences two days later on Thursday. Since this rule only applies to common years, we need to know what kind of year (common or leap) both Y and Y+1 are.

(4) BaTU-ThaKPaT (ב, ט"ו, תקפ"ט). If the molad at the **end** (i.e. around 29^{th} Elul) of a leap year, occurs on a Monday (יום ב) at or after 15:0589 (hh:pppp), JMT, the new year (i.e. the year following the leap year) commences on the next day, Tuesday. Since this only applies when the year that is ending is a leap year, in order to know if this will affect the commencement of Y or Y+1, we need to know what kind of year (common or leap) Y−1 and Y are.

REASONS FOR POSTPONEMENT RULES 3 AND 4

3. GaTRaD: Why must a common year, Y, whose molad occurs on a Tuesday at or after 09:0204, commence on the following Thursday?

The purpose of this rule is to prevent rules 1 and 2 from having the undesirable consequence of causing year Y to have a length of 356 days, which is one day longer than the maximum length of a common year.

Let **Y** be a common year, let **M** be the molad of year Y and let $\mathbf{M_1}$ be the molad of the following year, **Y+1**. For Y to come under the GaTRaD rule, M must occur between Tue, 09:0204 and Tue, 17:1079, inclusive. (If M occurred after the upper limit, Y would not come under this rule but under the rule *YaCH*.) M_1 = M + E12.[1] Adding E12 to both of the above limits for M gives us the range of values for M_1, which is between Saturday, 18:0000 and Sunday, 02:0875, inclusive.

M	Tu, 09:0204	Tu, 17:1079
+ E12	4, 08,0876	4, 08,0876
= $\mathbf{M_1}$	Sa, 18:0000	Su, 02:0875

Whatever the value of M_1 may be within that range, Y+1 must begin on Monday. At the lower end of that range, the commencement of Y+1 is postponed by *YaCH-ADU* from Saturday to Monday. At the upper end of that range, the commencement of Y+1 is postponed by *ADU* from Sunday to Monday. Since Y+1 begins on Monday, Y must end on Sunday.

Let us assume for the moment that Y may begin on Tuesday, the day of its molad. If it ends on a Sunday, as determined above, Y will have a length of 50 weeks, 6 days. This must be reduced by one, two or three days because the length of a common year must be 50 weeks plus 3, 4 or 5 days. Let us examine the ways by which Y might be reduced to a permitted length. We could make it:

- end one day earlier, on Saturday, making Y 50 weeks, 5 days. This is impossible because Y+1 would then begin on a Sunday, which is forbidden by the *ADU* rule.
- end two days earlier, on Friday, making Y 50 weeks, 4 days. Y+1 would then begin on Saturday. But this cannot be allowed: At the lower end of the range for M_1, the molad value requires the commencement of Y+1 to be postponed by *YaCH-ADU* from Saturday to Monday as mentioned above. At the upper end of the range for M_1, if Y+1 commenced on Saturday, its first day would precede the day of its molad (Sunday), which is never allowed.
- end three days earlier, on Thursday, making Y 50 weeks, 3 days. Y+1 would then begin on a Friday. But this is also impossible: Not only is this forbidden by *ADU*, its first day would precede the day of its molad (Saturday or Sunday), which is never allowed.

Since we cannot reduce the length of Y by making it end earlier, we must do so by postponing its commencement. It cannot be postponed by only one day because this would cause it to begin on a Wednesday, which is forbidden by *ADU*. It must therefore be postponed by two days. It will then begin on Thursday, end on Sunday and it will have a length of 50 weeks plus 4 days. It will be a regular common year.

To complete this explanation we should show why *GaTRaD* does not apply to a leap year.

If Y is a leap year M_1 = M + E13.[2] The full range of values for M compatible with Y beginning on a Tuesday is Mon, 18:0000 to Tue, 17:1079, inclusive. At the lower end of that range the commencement of Y is postponed by *YaCH* from Monday to Tuesday. At the upper end there is no postponement and Y begins on Tuesday, the day of its molad. Adding E13 to both of those limits gives us the range of values for M_1, which is Sun, 15:0589 to Mon, 15:0588, inclusive.

M	Mo, 18:0000	Tu, 17:1079
+ E13	5, 21,0589	5, 21,0589
= $\mathbf{M_1}$	Su, 15:0589	Mo, 15:0588

At the lower end of that range the commencement of Y+1 is postponed by *ADU* from Sunday to Monday. At the upper end of that range there is no postponement and Y+1 begins on Monday, the day of its molad. So whatever value M_1 may have within that range, Y+1 will begin on Monday and Y will have a length of 54 weeks, 6 days (384 days), which is a permitted length for a leap year. It will be a regular leap year.

1) **E12** is 4 days, 8 hours, 876 parts, the amount by which twelve calendric lunations exceeds 50 whole weeks.

2) **E13** is 5 days, 21 hours, 589 parts, the amount by which thirteen calendric lunations exceeds 54 whole weeks.

Whenever the commencement of a year, Y, is postponed by *GaTRaD*, the preceding year, Y−1, must end on a Wednesday. Only four year types end on a Wednesday: types Mon 353, Sat 355, Sat 383 and Thu 385. Of those, only two fit the calendric properties for year Y−1. They are Sat 355 and Thu 385. This is shown by the following calculations of the range of values for the molad (M_0) of Y−1, for a common year and for a leap year. In both cases, the commencement of Y−1 is postponed by *ADU*.

M	Tu, 09:0204	Tu, 17:1079
+ 7d [3]	10, 09:0204	10, 17:1079
− E12	4, 08,0876	4, 08,0876
= M_0	Fr, 00:0408	Fr, 09:0203

M	Tu, 09:0204	Tu, 17:1079
+ 7d [3]	10, 09:0204	10, 17:1079
− E13	5, 21,0589	5, 21,0589
= M_0	We, 11:0659	We, 20:0490

4. **BaTU-THaKPaT:** Why must a year, Y, whose molad occurs on a Monday at or after 15:0589, commence on the next day, Tuesday, if Y−1 is a leap year?

The purpose of this rule is to prevent rules 1 and 2 from having the undesirable consequence of causing the leap year Y−1 to have a length of 382 days, which is one day shorter than the minimum length of a leap year.

Let **Y−1** be a leap year so that **Y** may be subject to *BaTU-THaKPaT*. (Since two leap years never occur consecutively, Y must be a common year.) Let **M** be the molad of year Y and let the molad of the previous year, Y−1, be **M_0**. For Y to come under the rule *BaTU-THaKPaT*, M must occur between Mon, 15:0589 and Mon, 17:1079, inclusive. (If M occurred after the upper limit, Y would not come under this rule but under the rule *YaCH*.) M_0 = M − E13.[4] Subtracting E13 from both of the above limits for M, gives us the range of values for M_0.

M	Mo, 15:0589	Mo, 17:1079
+ 7d [3]	9, 15:0589	9, 17:1079
− E13	5, 21,0589	5, 21,0589
= M_0	Tu, 18:0000	Tu, 20:0490

M_0 is between Tue, 18:0000 and Tue, 20:0490, inclusive. Whatever the value of M_0 may be within that range, it will cause the commencement of Y−1 to be postponed by *YaCH-ADU* from Tuesday to Thursday. A Leap year has a length of 54 weeks plus 5, 6 or 7 days. Therefore, since Y−1 begins on Thursday, it must end on a Monday, Tuesday or Wednesday.

Tuesday is excluded because Y would then begin on Wednesday, which is forbidden by *ADU*. Wednesday is also excluded because this would postpone the commencement of Y from Monday (the day of its molad) to Thursday, a postponement of three days, whereas the maximum postponement possible under these rules is only two days. The only remaining possibility is Monday. This would give Y−1 a length of 54 weeks plus 5 days (383 days) and postpone the commencement of Y by one day from Monday to Tuesday. Y−1 will be a short leap year. (And Y will be of type Tue, 354.)

To complete this explanation we should show why *BaTU-THaKPaT* does not apply when Y−1 is a common year.

In such a case, M_0 = M − E12.[5] The full range of values for M compatible with Y beginning on a Monday is Sat, 18:0000 to Mon, 17:1079, inclusive. At the lower end of that range the commencement of Y is postponed by *YaCH-ADU* from Saturday to Monday. At the upper end there is no postponement and Y begins on Monday, the day of its molad.

Subtracting E12 from both of those limits for M gives us the range of values for M_0, which is Tuesday, 09:0204 to Thursday, 09:0203, inclusive. At the lower end of that range the commencement of Y−1 is postponed by *GaTRaD* from Tuesday to Thursday. At the upper end of that range there is no postponement and Y−1 begins on Thursday, the day of its molad. So whatever value M_0 may have within that range, Y−1 will begin on Thursday, and, since it ends on Sunday, it will have a length of 50 weeks plus 4 days (354 days), which is a legitimate length. It will be a regular common year.

M	Sa, 18:0000	Mo, 17:1079
+ 7d [3]	7, 18:0000	9, 17:1079
− E12	4, 08,0876	4, 08,0876
= M_0	Tu, 09:0204	Th, 09:0203

3) When subtracting from M, if it helps to simplify the arithmetic, we first add 7 days to the value of M and then perform the subtraction. The addition of 7 days to M does not change either its weekday or its time.

4) **E13** is 5 days, 21 hours, 589 parts, the amount by which thirteen calendric lunations exceeds 54 whole weeks.

5) **E12** is 4 days, 8 hours, 876 parts, the amount by which twelve calendric lunations exceeds 50 whole weeks.

EXPLANATION OF THE FORBIDDEN AND PERMITTED YEAR TYPES IN TABLE 2

From Table 1 showing the six possible year lengths and the explanation of year types in the paragraph preceding it, we saw above that a year type may be fully specified by two properties of a year – the weekday (**W**) on which it commences and its length (**L**) in days. We also saw that a common year may have 353, 354 or 355 days, i.e. 50 weeks plus 3, 4 or 5 days, and a leap year may have 383, 384 or 385 days, i.e. 54 weeks plus 5, 6 or 7 days.

In Table 2, the row-headings in column 1 are the four weekdays (W) on which a year may commence. (The other three are excluded by postponement rule *ADU*.) The headings of the other six columns are the six possible year lengths (L). There are 24 possible combinations (4 × 6) for these two properties of a year, and, for each such potential year type, W, L, there is a cell in the body of Table 2 where the row for W and the column for L intersect. Ten of those cells are shaded, indicating that those ten year types cannot occur. The following paragraphs explain why.

If a year, Y, is of type W, L, where L consists of n weeks + **d** days, then year Y would end on weekday W+d−1. That weekday appears as a two-letter abbreviated weekday name in the cell for that potential year type, W, L. The next weekday, W+d, is the weekday on which year Y+1 would commence. To determine whether a year of type W, L may occur, we first observe that if weekday W+d is one of the three weekdays on which a year may not commence because of the rule *ADU*, a year of type W, L may not occur. Of the ten shaded cells in Table 2, nine are excluded for this reason. The tenth one is year type Tuesday, 385. Such a year would end on a Monday. It is legitimate for a year to commence on the following weekday, Tuesday, so year type Tuesday, 385 must be forbidden for a different reason.

Let Y be a year of the type Tuesday, 385. It ends on a Monday, so year Y+1 must commence on a Tuesday. A year of 385 days is a leap year. Since two leap years cannot occur consecutively, Y+1 must be a common year. From Table 2 we see that the only common year that may commence on a Tuesday is a regular year of 354 days. So the question that now remains to be answered is why a year of type Tuesday, 354 cannot be preceded by a year of type Tuesday, 385. To answer this question we need to examine the possible molad values for years Y and Y+1.

Let **M** be the molad of Y and let $\mathbf{M_1}$ be the molad of the following year, Y+1. Since Y is a leap year of 13 months, to obtain the value of M_1, we add **E13** to M. For year Y to commence on a Tuesday, its molad, M, must occur between Monday, 18:0000, JMT (noon on Monday) and Tuesday, 17:1079, JMT (1 part before noon on Tuesday), both inclusive. Proof: What if M occurred outside this range? If M occurred on Monday before the lower limit, Y would commence on that Monday. If it occurred on Tuesday after the upper limit, then Y would be subject to postponement *YaCH-ADU* and would commence on Thursday. On the other hand, if M occurs within the above range, then at the lower end of that range, Y would be subject to postponement *YaCH* and will commence on Tuesday. At the upper end of the range, there is no postponement and Y will commence on the day of its molad, Tuesday. Adding E13 to both of these limits for M gives us the limiting values for the range within which M_1 must fall. They are Sunday, 15:0589 and Monday, 15:0588.

M	Mon, 18:0000	Tue, 17:1079
+ E13	5, 21,0589	5, 21,0589
= $\mathbf{M_1}$	Sun, 15:0589	Mon, 15:0588

At whatever time M_1 may occur within those limits, year Y+1 must commence on a Monday. If M_1 occurs at any time on Sunday, the commencement of Y+1 will be postponed by *ADU* to Monday. If M_1 occurs any time on Monday up to and including the upper limit, no postponement applies and Y+1 will commence on the day of its molad, Monday.

Thus, when we consider the molad values, we find that following any leap year, Y, that commences on a Tuesday, Y+1 must commence on a Monday, so Y must end on a Sunday. Therefore the length of Y must be 54 weeks + 6 days, which is 384 days. This is why a year of type Tuesday, 385 (a year length of 55 weeks) cannot occur. After eliminating this tenth forbidden year type, there are 14 remaining year types in Table 2.

When a similar exercise is applied to each of those 14 year types, the results show that there is no impediment such as this that would prevent any of those year types from occurring. So all of the 14 remaining year types are permissible. In the interest of brevity that proof is not included here.

EXPLANATION OF FORMULA (235Y−234) FOR NUMBER OF MONTHS UP TO YEAR Y

12(Y−1) and (7(Y−1)+1)\19 give, respectively, the number of ordinary and intercalated months up to Y. The latter is based on the fact that (7Y+1)\19 gives the number of leap years up to and including year Y.

ANNOUNCING THE MOLAD

The first day of a month is a minor festival called Rosh Chodesh. If the previous month is a long month, Rosh Chodesh is observed for two days commencing on the 30th of the previous month, because, since the duration of a lunation is only 29.53 days, it is likely that the next lunation begins some time during that 30th day. On the Shabbat preceding Rosh Chodesh, the synagogue service includes an announcement of when during the following week Rosh Chodesh will be observed. This is followed by a prayer (Birkat HaChodesh) that the coming month be a blessed one for us. In some synagogues it is customary to precede the announcement of Rosh Chodesh by stating exactly when the molad occurs. There are two reasons for this custom:

(a) It parallels to some degree the practices for regulating the older, observation-based calendar that preceded the calendar in use today, and at the same time it also contrasts that with present-day practice. That older calendar was regulated by monthly observations of the Moon's first reappearance after each astronomical New Moon. Observers were on the lookout for that event from sunset at the end of the 29th of the month. If they saw it, they would testify to this the next day at a court convened for that purpose in Jerusalem, and, based on that testimony, the court would proclaim that day the first of the next month. (If no witnesses appeared, the following day would be proclaimed as the first of the month in any event.) When that calendar evolved into the fixed-arithmetic calendar in use today, the method for regulating it changed from observation of the real Moon to the molad calculations of the mean Moon, as described above, so the old practice is now replaced by stating the molad and then announcing when Rosh Chodesh will occur.

There is a theory that the desire for this contrast was originally a reaction to Karaite practice. The Karaites continued the old practice of setting the calendar by real Moon sightings, and the synagogue announcements of the molad explicitly differentiated Rabbanite practice from that of the Karaites.

(b) The molad announcements also serve a practical purpose: Anyone familiar with the rules of the calendar (and such knowledge was considered fundamental to a good Jewish education) could calculate the calendar of any year, should they ever need to, from the molad value stated in any recent synagogue announcement, as explained above.

For this purpose and also for other reasons it is important that the molad value is announced in the format and in the time units in which the calendar rules and the molad values associated with them are traditionally expressed. That is, the time should be announced in Jewish Mean Time, using 24-hour clock notation, and as hh:pppp (hours and parts only). This means that hh = 0 to 23, where zero hours is at mean sunset (18:00 hours, civil time), and pppp = 0 to 1079 parts of an hour. Alternatively, the time could be announced as hh:mm:pp, where hh = 0 to 23 hours, mm = 0 to 59 minutes and pp = 0 to 17 parts, but this is not recommended as it does not conform to the traditional format.

It is preferable to state the molad weekday in Hebrew because the Jewish day commences six hours before the corresponding civil weekday so that only the *last* 18 hours of the Jewish day and the *first* 18 hours of the corresponding civil day coincide, as shown in the diagram above. For clarity, it is best to also state explicitly that the day and the hour number are not counted from midnight but from when the Jewish calendar day begins, which is at mean sunset, exactly six hours earlier.

It should be obvious from the above paragraph that it is absolutely wrong to convert the molad time to civil time by subtracting six hours. This only introduces ambiguity as to what time the molad actually occurs and 25% of the time it will result in the wrong molad weekday being stated. It also makes it difficult to determine from the molad value (as described above) if one of the postponement rules applies. Likewise, never adjust a molad time for Daylight Saving Time. Doing so will not only introduce ambiguity as to the molad time, it will sometimes result in the wrong molad weekday being stated.

Yaaqov Loewinger of Israel published an essay in Hebrew in the Summer 2008 issue of *Hakirah, The Flatbush Journal of Jewish Law and Thought* on the subject of the announcement of the molad in synagogues. It is a good review and cites an impressive collection of traditional sources on the subject. The format recommended by Loewinger is the one outlined above. Please read his essay here.

Everything stated above in relation to the correct format for stating the molad in the synagogue announcements also applies to how the moladot should be specified in published calendars that include that information. I recommend publishing them as per this example.

For further reading on this subject, see the Wikipedia entry Molad, and this thread on the calendar discussion group Caldnr-L of erroneous practices in publishing and announcing moladot.